

Fourth Semester B.E. Degree Examination, Dec.2013/Jan.2014
Signals and Systems

Time: 3 hrs.

Max. Marks:100

**Note: Answer FIVE full questions, selecting
at least TWO questions from each part.**

PART – A

- 1 a. Obtain the even and odd components of the following signals: i) $x(n) = \{1, 2, 3\}$; ii) $x(t)$ as shown in Fig.Q.1(a). (04 Marks)
- b. Categorize each of the following signals as energy or power signals and find the energy or power of the signal: i) $x(t)$ as shown in Fig.Q.1(b); ii) $x(n) = \left(\frac{1}{3}\right)^n u(n)$. (06 Marks)
- c. For the signal $y(t)$ shown in Fig.Q.1(c), sketch $y(t-3)$, $y(-t)$, $y(-t+3)$ and $y(2t+4)$. (05 Marks)
- d. Test the system $y(n) = 2x(n) \times x(n)$ for memory, causality, linearity, stability and time invariance (Note: $x(n)$ is the input). (05 Marks)

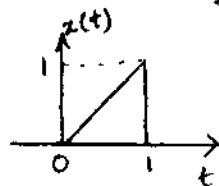


Fig.Q.1(a)

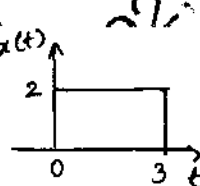


Fig.Q.1(b)

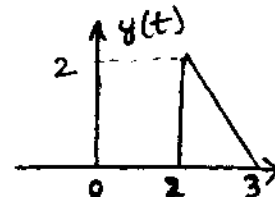


Fig.Q.1(c)

- 2 a. Evaluate the continuous-time integral $y(t) = x(t) * h(t)$ with $x(t) = e^{-2t} (u(t) - u(t-3))$ and $h(t) = 2(u(t+1) - u(t+4))$. (12 Marks)
- b. Obtain the discrete-time convolution sum given the impulse response $h[n] = \{1, 4, 3, 8\}$ and input $x[n] = \{2, 5, 7\}$. (08 Marks)

- 3 a. Compute the step response of the system with impulse response $h(t) = e^{-3t} u(t)$. (04 Marks)
- b. Evaluate the natural response of the system described by

$$\frac{d^2 y(t)}{dt^2} + \frac{2dy(t)}{dt} + y(t) = \frac{d}{dt} x(t); \quad y(0) = 1; \quad \frac{dy(t)}{dt} \Big|_{t=0} = 1.$$

Draw the direct form I and II realization for an LTI system described by the difference equation $y(n) + 2y(n-1) - 3y(n-3) = x(n) - 2x(n-2)$. (08 Marks)

- 4 a. Determine the Fourier series representation for the signal $x(t)$ given by $x(t) = 5 \cos\left(\frac{\pi t}{2} + \frac{\pi}{6}\right)$. Sketch the magnitude and phase spectrum. (06 Marks)
- b. State and prove the time-shift property of the Fourier series. (05 Marks)
- c. Given the DTFS $x(k) = \{1, 2, -j, 0.5\}$ of a periodic signal $x[n]$ with period $N = 7$, find $x(6)$ and $x(9)$ using suitable property. (04 Marks)
- d. Determine $x(t)$ given $\omega_0 = 3\pi$ and $x(k) = j\delta(k-1) - j\delta(k+1) + \delta(k-3) + \delta(k+3)$. (05 Marks)

PART - B

- 5 a. Find the Fourier transform of $x(t) = e^{-3t} u(t)$. (05 Marks)
 b. Explain Parseval's relationship for the Fourier transform. (04 Marks)
 c. Using inverse Fourier transform, find $x(t)$ whose $X(j\omega) = \frac{-j\omega}{(j\omega)^2 + 3j\omega + 2}$. (06 Marks)
 d. Compute the DTFT of $x[n] = \left(\frac{1}{2}\right)^n u(n)$. (05 Marks)

- 6 a. Obtain the impulse response of the system described by the differential equation $\frac{2dy(t)}{dt} + 3y(t) = 7x(t)$ using Fourier transform. (06 Marks)
 b. Explain the sampling theorem for lowpass signals. (04 Marks)
 c. Find the Fourier transform of the discrete-time signal $x[n]$

$$x[n] = \left(\frac{1}{2}\right)^n u(n). \quad (05 \text{ Marks})$$

- d. Determine the difference-equation description for the system with the frequency response $H(e^{j\Omega}) = \frac{1 + e^{-j\Omega}}{3 + e^{-j2\Omega}}$. (05 Marks)

- 7 a. Compute the Z-transform and ROC of the following signals:
 i) $x(n) = 2\delta(n+1) + 4\delta(n-2)$; ii) $y(n) = 3^n u(-n)$. (06 Marks)
 b. State and prove the time-shifting property of the Z-transform. (05 Marks)

- c. Find the inverse Z-transform of $X(z) = \frac{\left(\frac{1}{4}\right)z^{-1}}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - \frac{1}{4}z^{-1}\right)}$; ROC $\frac{1}{4} < |z| < \frac{1}{2}$. (06 Marks)

- d. Using power series expansion, find (five terms of) $x(n)$ given its $X(z) = \frac{z}{2z^2 - 3z + 1}$; $|z| < \frac{1}{2}$. (03 Marks)

- 8 a. Given $H(z) = \frac{1}{1 - 3z^{-1}}$; ROC: $|z| > 3$. Is the system stable? Causal? Give reasons. (04 Marks)

- b. Find the difference equation description for a system with transfer function $H(z) = \frac{5z + 3}{z^2 + 2z + 4}$. (05 Marks)

- c. Compute the natural response, forced response and total response of the LTI system with difference equation $y(n) + 3y(n-1) = x(n) + x(n-1)$ if the input is $x(n) = \left(\frac{1}{2}\right)^n u(n)$ and $y(-1) = 2$ is the initial condition. (11 Marks)
